Exercises with matrices

Manuel Lleonart Anguix

1 Operations with matrices

Exercise 1. *Let be the matrices*

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 4 \\ 6 & 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E = \begin{pmatrix} 1 & 3 & 4 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$$

1. *What is the size of the previous matrices?*
2. *Can you name the types or characteristics of those types as defined in section L 2.3?*

Exercise 2.

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 4 \\ 6 & 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 8 & 1 \\ 5 & 5 & 4 \\ 9 & 2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 8 & 7 \\ 1 & 8 & 3 \end{pmatrix}$$

Do the following operations.

1. $A + B$.
2. $A - C$.
3. $2A + B - 3C$.
4. $A \cdot B$.

5. $B \cdot C$
6. $A \cdot B \cdot C$.
7. $|A|$, $|B|$ i $|C|$.

Calculating inverses:

1. Calculate A^{-1} by Gauss.
2. Calculate B^{-1} using the adjoint matrix.

Exercise 3.

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 8 \\ 5 & 8 \\ 10 & 16 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 8 & 7 \\ 1 & 8 & 3 \end{pmatrix}$$

Calculate, if you can, the following matrices. If you can't, explain why.

1. $A + B$, $A + C$.
2. $A \cdot B + C$, $B \cdot A + C$.
3. ABC, BAC.
4. $|A| + |B|$.
5. A^2, C^2.
6. C^{-1}.
7. $rank(A)$, $rank(B)$, $rank(C)$.
8. A^T, B^T *i* C^T.
9. *Is some of the previous matrices symmetric? Which condition must a matrix meet to be symmetric?*

Exercise 4. *Answer the following questions*

- *What is the maximum range of a matrix* $m \times n$*?*
- *How can we compute the range?*
- *Write all the minors of size two of the matrix* C *on the previous exercise.*

Exercise 5. *Let the vector b be*

$$b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

- *Can you define a matrix representing the following linear application ϕ? What will be the dimension of the matrix? Why?*

$$\phi(b) = \begin{pmatrix} 2b_1 - b_2 + 3b_3 \\ b_2 - b_1 + 4b_3 \\ 5b_1 - b_2 + b_3 \end{pmatrix}$$

Exercise 6.

$$A = \begin{pmatrix} 3 & 2 & 1 & 3 \\ 1 & 2 & 4 & 3 \\ 1 & 6 & 3 & 5 \\ 1 & 1 & 1 & 3 \end{pmatrix}$$

1. *Calculate the determinant of A developing it by rows (or columns).*
2. *Calculate the determinant of A using equivalent matrices.*

Exercise 7. * *Let X some square matrix such that*

$$X^2 + X + I = 0$$

- *What can we say about its determinant?*
- *And about the invertibility of X? Why?*
- *Can you express the inverse of X as a function of X?*

Exercise 8. * *Let C be as in exercise 3.*

$$C = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 8 & 7 \\ 1 & 8 & 3 \end{pmatrix}$$

If we have the system

$$C^2 + CX + I = 0$$

What should be the dimensions of X? Compute the value of X.

Exercise 9. *Be the matrices*

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 1 & 2 \\ 5 & 4 & 4 \end{pmatrix}$$

Without performing any operation, answer.

1. *Calculate $|A|$ i $|B|$.*
2. *Calculate $|AB|$.*
3. *Is true that $|AB| = |BA|$?*
4. *Is true that $AB = BA$?*

Exercise 10. *Let*

$$v = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}, \quad w = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}, r = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

- *Can you express r as a linear combination of v and w? Why? Why not?*
- *Is the set of these three vector linearly independent? Why? Why not?*
- *Let A be a matrix with columns v, w and r (in that order). Is A singular? What is the rank of the matrix? What happens if we change the order of the columns?*

Exercise 11. *

1. *Calculate the eigenvalues of matrices in exercise 2.*
2. *Calculate its trace.*
3. *Are those matrices positive definite?*